Statistical MIMO Radar

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Abstract Inspired by recent advances in multiple-input multiple-output (MIMO) communications, we introduce the statistical MIMO radar concept. Unlike beamforming, array radar, or STAP, which presuppose a high correlation between signals either transmitted or received by an array, the proposed MIMO radar exploits the independence between signals at the array elements. Whereas correlation-based array techniques are capable of providing degrees of freedom for spatial filtering, they have no bearing on the effects of target scattering. Radar targets generally consist of many small elemental scatterers that are fused by the radar waveform and the processing at the receiver to result in echoes with fluctuating amplitude and phase. In conventional radar, target radar cross-section (RCS) fluctuations are regarded as a nuisance that degrades radar performance. The novelty of statistical MIMO radar is that it takes the opposite view, namely, it capitalizes on target RCS scintillations and glint to improve the radar's performance. MIMO radar utilizes multiple antennas at both the transmitter and receiver. It can be applied in monostatic or bistatic modes. The antennas at each end of the radar system have to be sufficiently separated such that the target provides uncorrelated reflection coefficients between each transmit/receive pair of antennas. We demonstrate that the MIMO radar greatly improves detection and estimation performance due to the absence of target fades. Specifically, statistical MIMO radar overcomes target RCS fluctuations by averaging over many decorrelated channels between transmit and receive antennas. Subsequently, the received signal is a superposition of independently faded signals, and the average SNR of the received signal is more or less constant. This is equivalent to converting a Swerling case I RCS to a Swerling case II, but without the loss of time. Moreover, MIMO spatial diversity also eliminates the deep interference nulls in the elevation coverage due to surface multipath reflection.

Report Documentation Page

Form Approved
OMB No. 0704-0188

Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

1. REPORT DATE **20 DEC 2004**	2. REPORT TYPE **N/A**	3. DATES COVERED **-**
4. TITLE AND SUBTITLE **Statistical MIMO Radar**		5a. CONTRACT NUMBER
		5b. GRANT NUMBER
		5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)		5d. PROJECT NUMBER
		5e. TASK NUMBER
		5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **New Jersey Institute of Technology; Lehigh University**		8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)		10. SPONSOR/MONITOR'S ACRONYM(S)
		11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES
See also, ADM001741 Proceedings of the Twelfth Annual Adaptive Sensor Array Processing Workshop, 16-18 March 2004 (ASAP-12, Volume 1)., The original document contains color images.

14. ABSTRACT

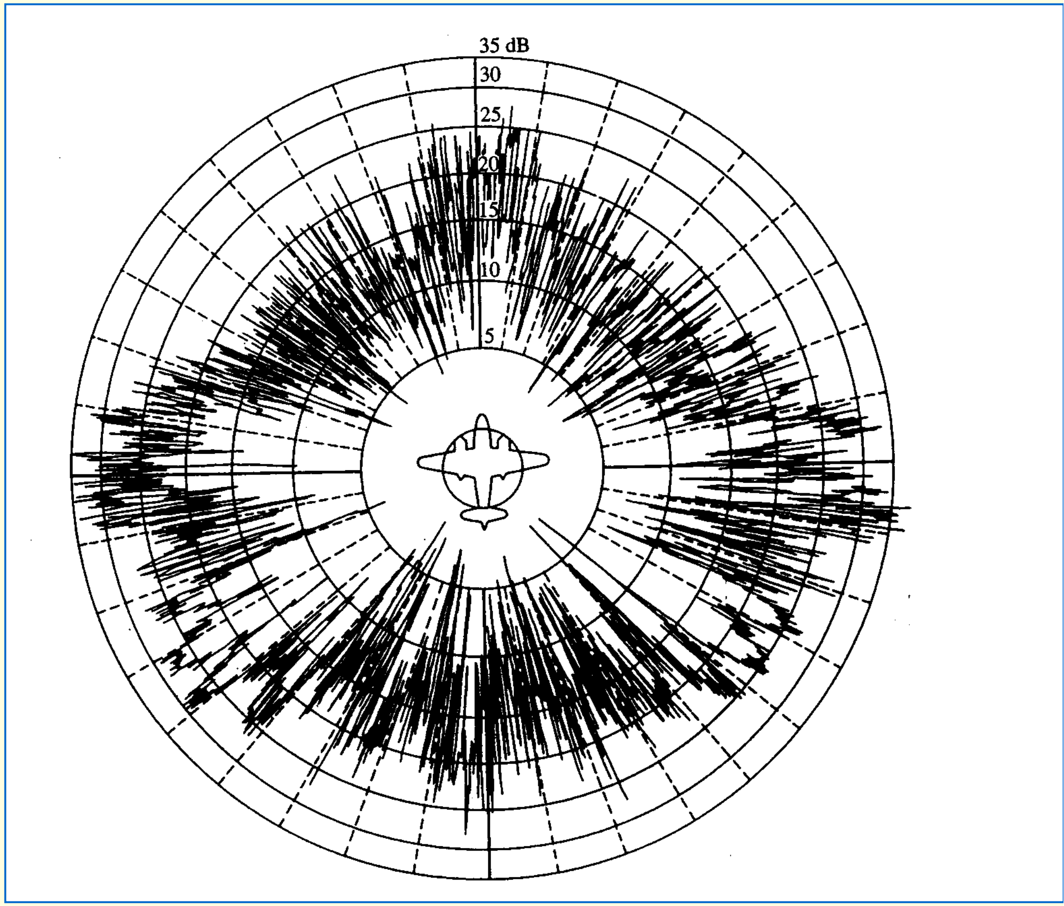
15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**UU**	**13**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

Motivation

- Radar targets provide a rich scattering environment.
- Conventional radars experience target fluctuations of 5-25 dB.
- Slow RCS fluctuations (Swerling I model) cause long fades in target RCS, degrading radar performance.
- In statistical MIMO the **angular spread** of the target backscatter is exploited in a variety of ways to extend the radar's performance envelope.



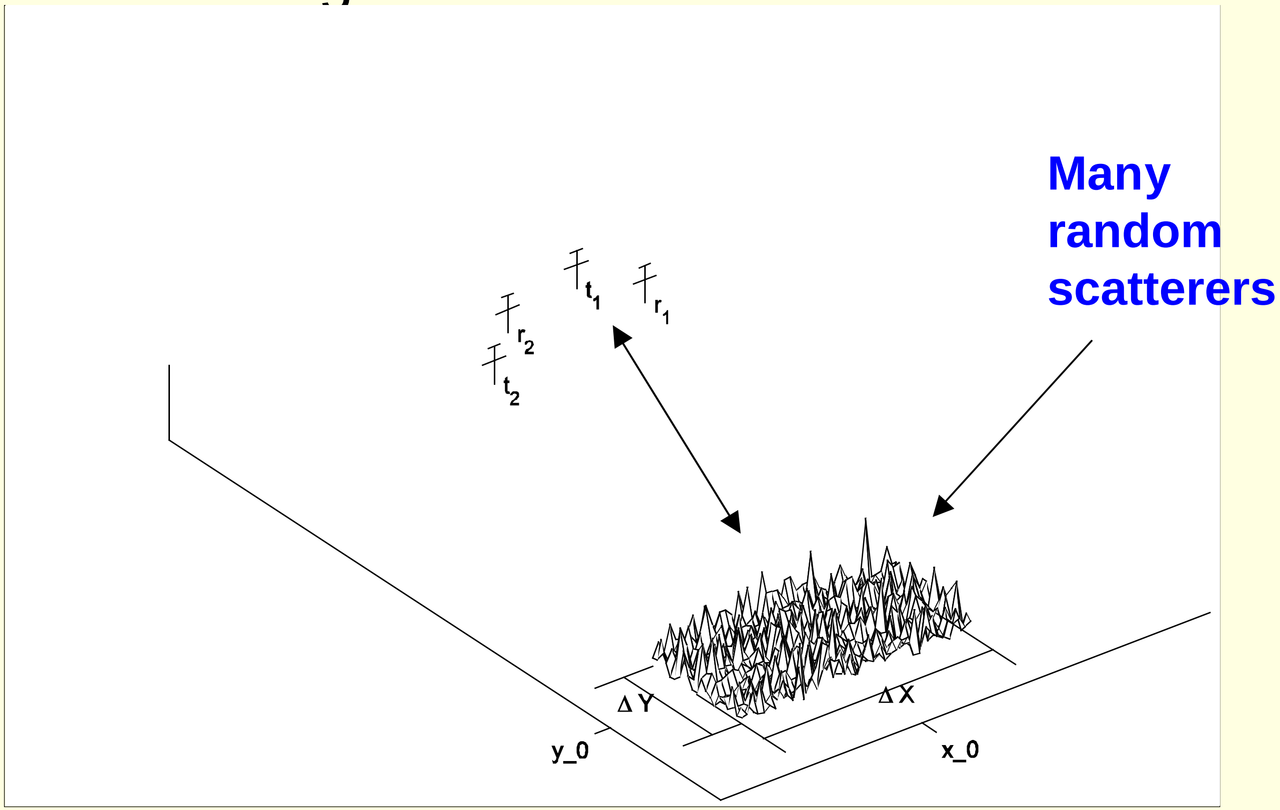
Backscatter as a function of azimuth angle, 10-cm wavelength [Skolnik 2003].

The S-MIMO Concept

- Statistical-MIMO radar offers the potential for significant gains:
  - Detection/estimation performance
  - Resolution performance
- Here, we focus only on detection performance
- Our results question the common belief that one should maximize the coherent processing gain.
- With S-MIMO a very sparse array of sensors transmits a set of orthogonal waveforms.
- By using this approach, we create many "independent" radars, that average out target scintillations.

Signal Model

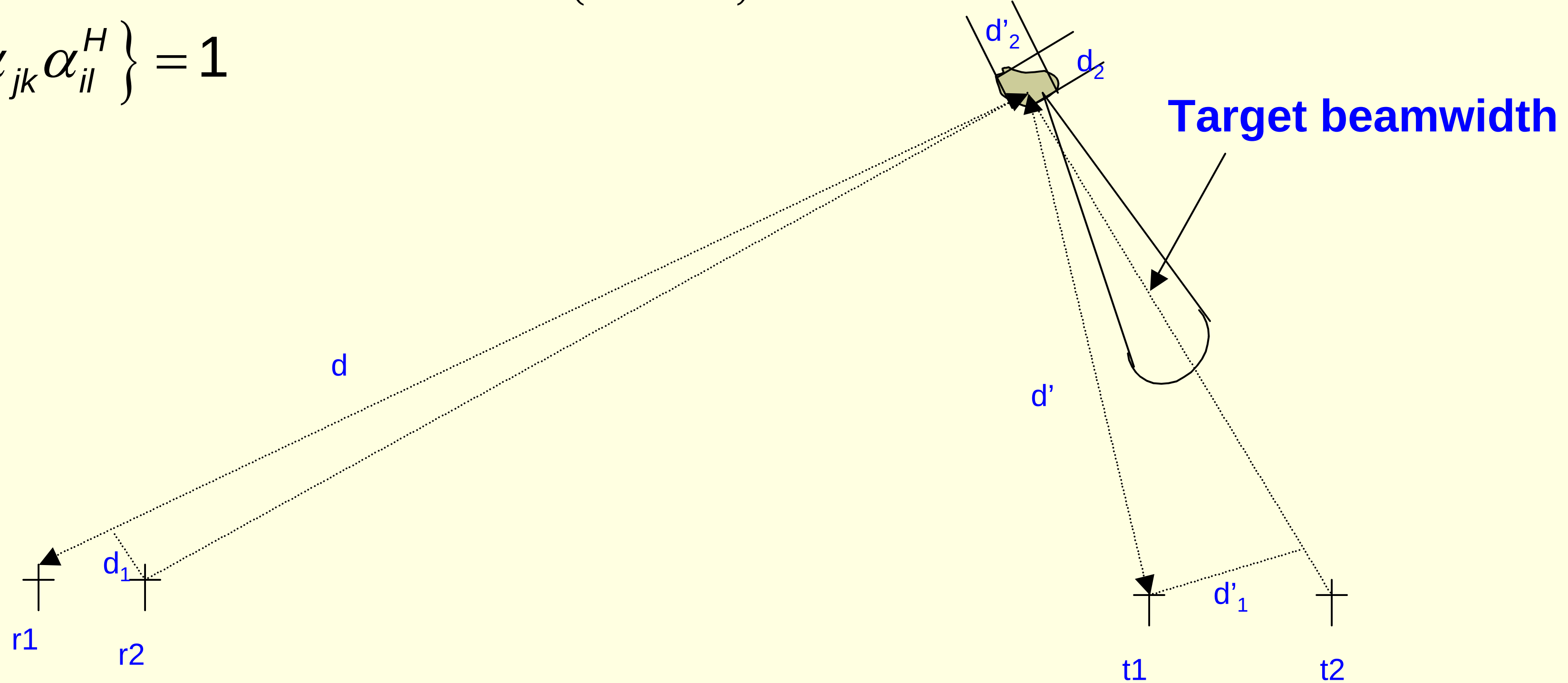
- Point source assumption dominates current models used in radar theory.
- This model is not adequate for an array of sensors with large spacing between the array elements.
- Distributed target model



Signal Model (Cont.)

- Denote by α_{jk} the gain between the *k*th transmitter and *j*th receiver. It can be shown that $\alpha_{jk} \sim CN(0,1)$.

- Take α_{jk} and α_{il}. We can show that if either $d_1 > d\lambda_c / d_2$ or $d'_1 > d'\lambda_c / d'_2$, then $E\left\{\alpha_{jk}\alpha_{il}^H\right\} \approx 0$, and otherwise $E\left\{\alpha_{jk}\alpha_{il}^H\right\} = 1$



Phased Array Radar

- Phased array radars consist of closely spaced sensors.
  The gain between each transmitter receiver pair is the same.

- Transmitted waveform is $\mathbf{s}(t)$

- This gives rise to the following received signal model

$$\mathbf{r}(t) = \sqrt{\frac{E}{M}} \alpha \mathbf{a}(x_0, y_0) \mathbf{b}(x_0, y_0)^H \mathbf{s}(t - \tau) + \mathbf{n}(t)$$

- If beamformer is applied at both the transmitter and the receiver, then the received signal at the output of the beamformer equals

$$y(t) = \sqrt{\frac{E}{M}} \alpha \left\| \mathbf{a}(x_0, y_0) \right\|^2 \left\| \mathbf{b}(x_0, y_0) \right\|^2 s(t - \tau) + n'(t)$$

S-MIMO Radar

- In S-MIMO radar, the inter element spacing is large. The gain between every transmitter receiver pair is different.

- The received signal is given by

$$\mathbf{r}(t)=\sqrt{\frac{E}{M}}\mathbf{Hs}(t-\tau)+\mathbf{n}(t) \qquad \mathrm{vec}(\mathbf{H})\sim \mathrm{CN}(\mathbf{0},\mathbf{I})$$

- Each transmitting element transmits one of M orthogonal waveforms.

- By matched filtering the received signal at each sensor with each of the transmitted waveforms we can reconstract

$$r_{ji}(t)=\sqrt{\frac{E}{M}}\alpha_{ji}s_i(t-\tau)+n_{ji}(t)$$

- Therefore, instead of coherent gain of *MN*, we created *MN* independent radars.

The Radar Detection Problem

- The radar detection problem:

  H_0 : Target does not exists at delay τ

  H_1 : Target exists at delay τ

- Assume that all the parameters are known. The optimal detector is the LRT detector, and it is given by,

$$T = \log \frac{f\left(\mathbf{r}\left(t\right) | H_1\right)}{f\left(\mathbf{r}\left(t\right) | H_0\right)} \mathop{\gtrless}_{H_1}^{H_0} \delta$$

S-MIMO Radar

- Denote by **x** the vector that contains the output of a bank of matched filters sampled at τ. The optimal detector is

$$T = \|\mathbf{x}\|^2 \mathop{\gtrless}_{H_0}^{H_1} \delta, \qquad \text{where} \quad \delta = \frac{\sigma_n^2}{2} F_{\chi^2_{2MN}}^{-1}\left(1 - P_{FA}\right)$$

- It is possible to compute the probability of detection as a function of the probability of false alarm, and it equals

$$P_D = 1 - F_{\chi^2_{2MN}}\left(\frac{\sigma_n^2}{\frac{E}{M}+\sigma_n^2} F^{-1}_{\chi^2_{2MN}}\left(1-P_{FA}\right)\right)$$

Phased Array Radar

- Let $x = \int \mathbf{r}^H(t)\mathbf{a}(x_0, y_0)s(t-\tau)dt$. The optimal detector:

$$T = |x|^2 \underset{H_1}{\overset{H_0}{\gtrless}} \delta \qquad \delta = \frac{N\sigma_n^2}{2} F^{-1}_{\chi^2_2}\left(1-P_{FA}\right)$$

$$P_D = 1 - F^{-1}_{\chi^2_2}\left(\frac{\sigma_n^2}{\sigma_n^2 + EN} F^{-1}_{\chi^2_2}\left(1-P_{FA}\right)\right)$$

The Invariance Detector

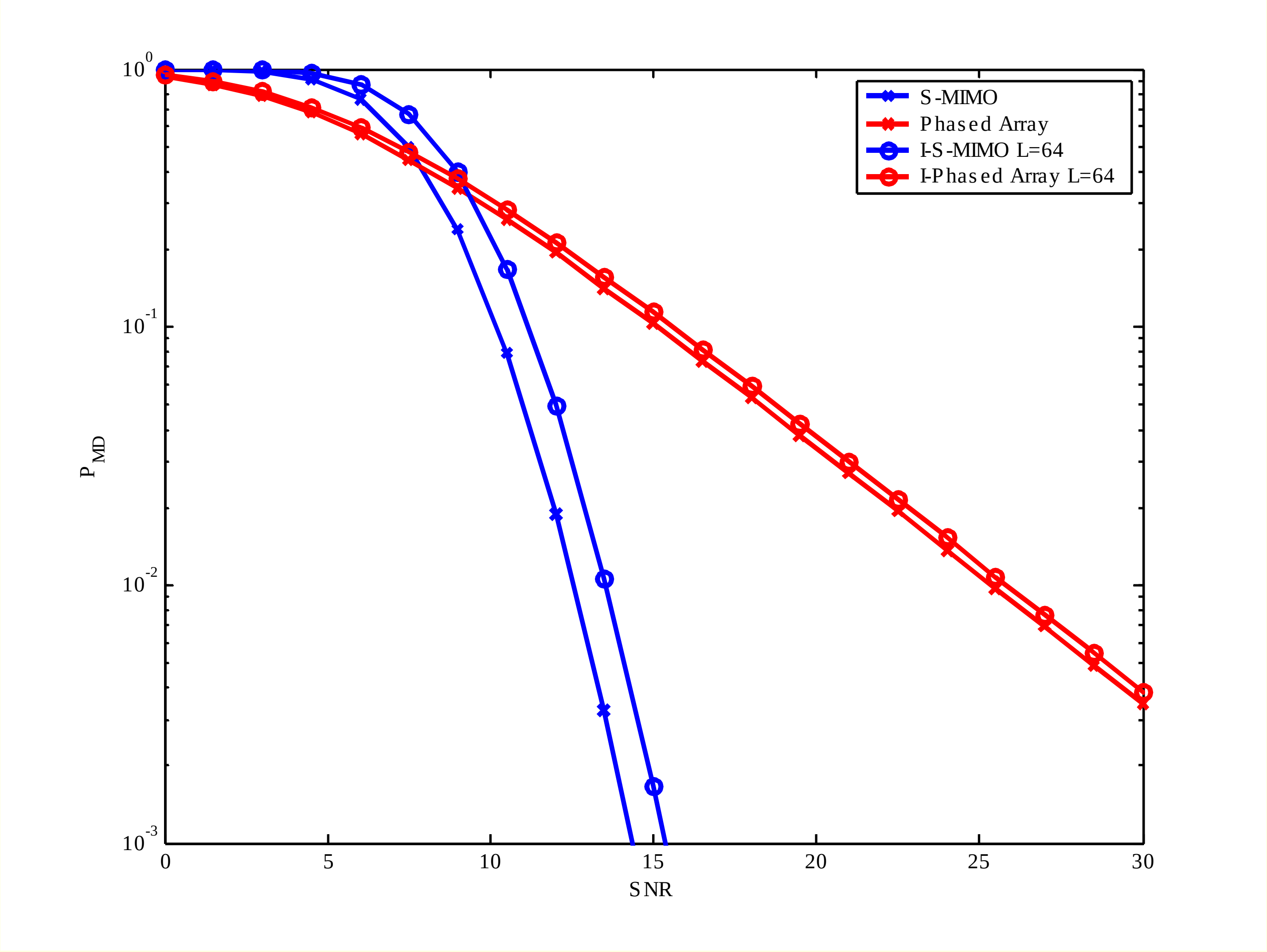
- Assume access to a vector **y** that contains L samples of the noise process.
- Note that $\|\mathbf{y}\|^2 / L$ is the ML estimate of the noise level.
- The optimal detector whose performance depends only on SNR (not on the noise level)

$$T = \frac{\|\mathbf{x}\|^2}{\|\mathbf{y}\|^2} \mathop{\gtrless}_{H_0}^{H_1} \delta$$

- This test statistic is very intuitive. It normalizes the UMP test by the best estimate of the noise level.

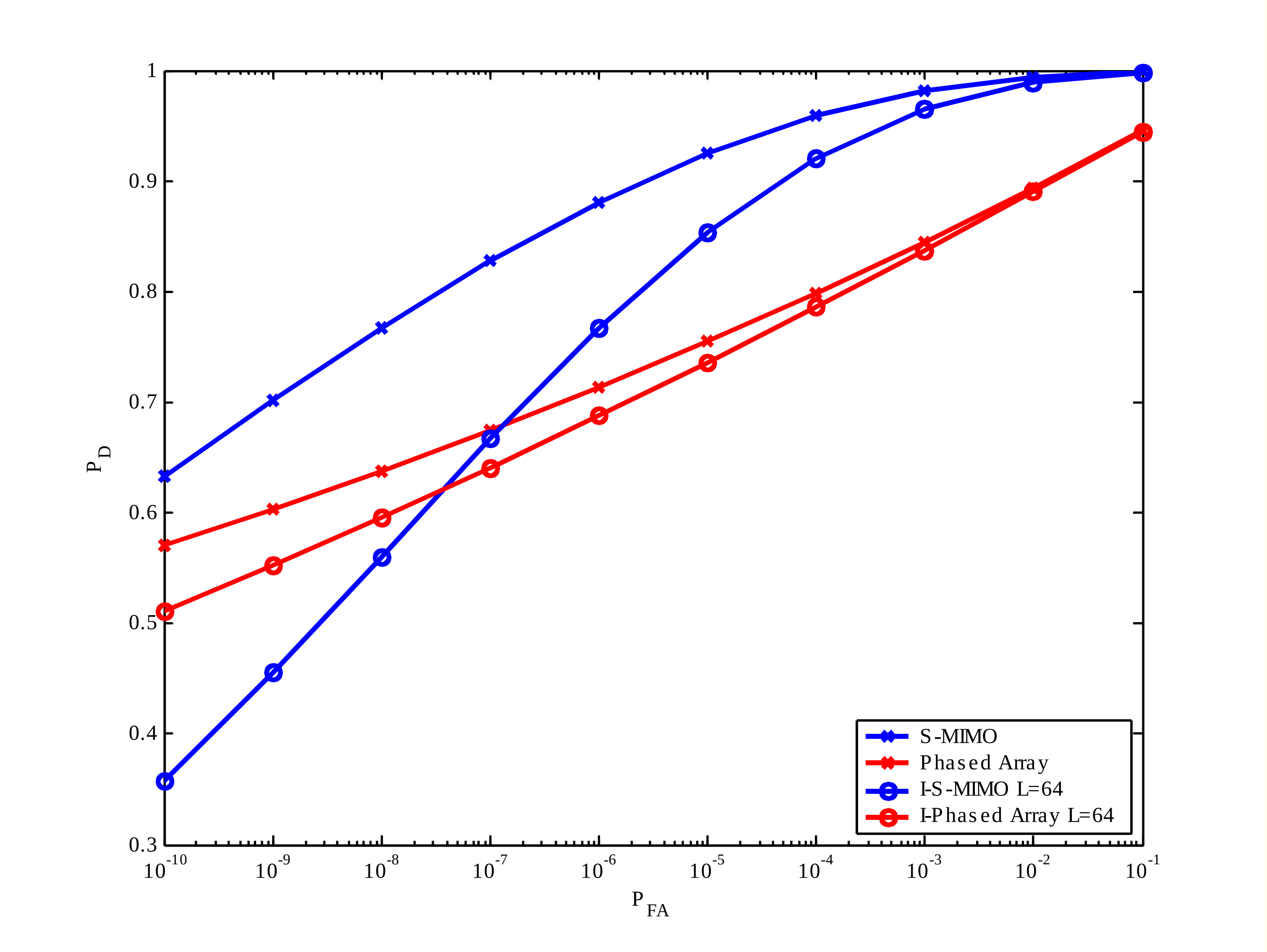
Example: Miss Probability

- Assume a system with four receiving and one or two transmitting antennas, M=2, N=4, and the probability of false alarm is 1e-6



Example: ROC

- The following figure depicts the ROC. SNR=10dB.



Concluding Remarks

- S-MIMO is a new concept for radar systems.
- This concept utilizes spatial diversity in order to overcome target scintillations.
- At 90% probability of detection, the proposed system outperform phased array radars by 5 dB, which is equivalent to almost twice the range.
- The S-MIMO radar can be shown to have superior performance in range estimation and resolution as well.